

Entanglement interpretation of black hole entropy in string theory

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ABSTRACT: We show that the entropy resulting from the counting of microstates of non extremal black holes using field theory duals of string theories can be interpreted as arising from entanglement. The conditions for making such an interpretation consistent are discussed. First, we interpret the entropy (and thermodynamics) of spacetimes with non degenerate, bifurcating Killing horizons as arising from entanglement. We use a path integral method to define the Hartle-Hawking vacuum state in such spacetimes and discuss explicitly its entangled nature and its relation to the geometry. If string theory on such spacetimes has a field theory dual, then, in the low-energy, weak coupling limit, the field theory state that is dual to the Hartle-Hawking state is a thermofield double state. This allows the comparison of the entanglement entropy with the entropy of the field theory dual, and thus, with the Bekenstein-Hawking entropy of the black hole. As an example, we discuss in detail the case of the five dimensional anti-de Sitter, black hole spacetime.

KEYWORDS: AdS-CFT Correspondence, Black Holes, Black Holes in String Theory.

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1. Introduction

The discovery that black holes have entropy [1] and that they radiate [2], has led to many speculations about their quantum gravity origin. The unusual non-extensive nature of the entropy of black holes and their effect on matter thrown into them, leads to conjectures that entropy bounds should exist [3]. The area scaling character of the entropy seems to imply a “holographic principle” [4, 5], where a system has an areas’ worth of physical degrees of freedom [6]. The celebrated AdS/CFT correspondence [7] is one such example. Yet, about 30 years after its discovery, solid information regarding the physical origin of black hole entropy is still lacking.

The entropy of a thermodynamic system is a measure of the number of the available microstates of the system that can not be discriminated by an observer who measures macroscopic quantities. For example, when considering black body radiation, one needs to count the number of the appropriate photon configurations. The entropy of this “photon gas” is the logarithm of the number of states in phase space that are available to the photons. From this perspective black hole entropy relies on some fundamental definition of microstates in a theory of quantum gravity: it is a measure of the number of microstates of the black hole that an observer would not be aware of when measuring macroscopic parameters.

String theory has been able to successfully count these microstates explicitly [8–11]. This has been done by appealing to a duality between gravitational systems and (conformal) field theories. Black holes are described in a low energy limit of a gravitating theory. They also have an equivalent description as a field theory, whereby their microstates may be counted explicitly. This method does not give physical insight regarding the nature of the microstates of the black hole nor does it offer a reason for its area scaling property. Recent work regarding counting of microstates from a different perspective can be found in [12–14].

The idea that we pursue is that the microstates are those due to entanglement of the vacuum of the black hole. Its main features are the observer-dependence of the entropy, the role of quantum correlations across causal boundaries, and non-extensiveness. In fact, we claim that this entanglement mechanism is not specific to black holes but to any spacetime with a bifurcating Killing horizon. A $d+1$ dimensional spacetime that has a $d-1$ dimensional space-like surface Σ , on which each point is a fixed point of an isometry, possesses a bifurcating Killing horizon. This is the null surface generated by the null geodesics emanating from Σ . It can be shown that the bifurcating Killing horizon is invariant under the isometry [15].

The vacuum state of fields (including fluctuations of the gravitational field) in a spacetime that has a bifurcating Killing horizon may be chosen so that the energy-momentum tensor does not diverge on the horizon. This vacuum state is called the Hartle-Hawking (HH) state. Examples of spacetimes with a single bifurcating Killing horizon are de Sitter space, Kruskal spacetime, the massive non-rotating BTZ black hole, and Minkowski space (in Rindler coordinates.)

As we shall show, one may view the HH state as an entangled state with respect to the region of space being observed and an inaccessible region beyond the horizon. Being restricted to a part of space, an observer will see a mixed rather than pure state. The entropy of such a mixed state is called entanglement entropy. In section 4, we argue, following the pioneering work of [16–18], that one can interpret the calculation of the number of states in field theory duals, for a wide class of spacetimes, as a measure of entanglement entropy.

We highlight the difference, in principle, between the entropy of a “microstate gas,” which conveys information regarding the quantum mechanical nature of microstates that “look like” a black hole, and the entanglement entropy, which is a measure of an observers lack of information regarding the quantum state of the system in an inaccessible region of spacetime.

Previously, there have been several attempts to relate the entropy and thermodynamics of the BH to entanglement. In some of these analyses the entanglement entropy was interpreted as a one-loop correction to the classical Bekenstein-Hawking entropy [19–23], while others, particularly using the “induced gravity” approach, aimed to account for the entire entropy of the black hole in terms of entanglement [24]. We argue by evaluating the BH entropy using a field theory dual, that the leading contribution to the entropy is due to entanglement. Our argument depends on the assumption that the entropy of the field theory dual is equal to the BH entropy, an equality that is usually a direct consequence of the duality. Our argument implies that the microstates of the black hole are due to the

entangled nature of the BH vacuum, and are a result of an observer’s inability to access the degrees of freedom that are hidden beyond the horizon.

Entanglement of the BH vacuum state was first considered a long time ago by Israel [25]. Here we generalize the path integral method of [26] to curved spacetimes with non-degenerate, bifurcating Killing horizons. We use our method to extend the results of [25]. Our analysis explicitly reveals the geometric nature of the vacuum state. This relationship between geometry and entanglement has been formally discussed in [27].

For the case of AdS BH’s, the argument that the BH entropy is due to entanglement follows from the results of Maldacena [18]. We discuss this case in detail and relate the divergence of the entanglement entropy to Newton’s constant G_N .

As we shall discuss in section 6, the entanglement interpretation simplifies some issues regarding area scaling, non-unitary evolution, and the information paradox. Although it leads to some results that can be obtained using the “membrane paradigm” [28], which describes the BH in terms of a stretched horizon and a thermal atmosphere of the Hawking radiation near the horizon, it is conceptually different.

In section 2, we review the concept of entanglement and entanglement entropy in a simplified EPR-like model. In section 3, we explain and review the entangled nature of the HH vacuum state in spacetimes with a bifurcating Killing horizon. In section 4, we discuss entanglement for spacetimes having a field theory dual. Section 5 deals with the AdS-BH. We conclude with a discussion of these results.

2. Review of entanglement entropy

In this section we review the concepts of entanglement and entanglement entropy in quantum mechanics together with the construction of the thermofield double state and its significance. This section establishes the notation for the rest of the paper.

We start from a simple, well known, example of an EPR type experiment: suppose we start with two spin half particles in a singlet state: $|0, 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2)$. We wish to find the state as seen by an observer who measures the spin of one of the particles. Starting with $|0, 0\rangle$, or $\rho = |0, 0\rangle\langle 0, 0|$, and tracing over the spin degrees of freedom of the second particle we find

$$\begin{aligned} \rho_1 &= \text{Trace}_2 \rho \\ &= {}_2\langle \uparrow | 0, 0\rangle\langle 0, 0 | \uparrow \rangle_2 + {}_2\langle \downarrow | 0, 0\rangle\langle 0, 0 | \downarrow \rangle_2. \end{aligned}$$

This gives

$$\rho_1 = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}.$$

The entropy of the state measured by the second observer will now be $S_1 = -\text{Tr} \rho_1 \ln \rho_1 = \ln 2$. Even though we started off with a pure state with no entropy, an observer restricting her measurements to part of the available Hilbert space will see a mixed state with non-zero entropy. This entanglement entropy S_1 is intrinsically observer dependant. The microstates

being counted are the, now uncorrelated, spin degrees of freedom. Obviously, this analysis is symmetric with respect to both Hilbert spaces.

Generally, one defines an entangled state (on a product of two Hilbert spaces) as a state, that under no local unitary transformation can be reduced to a direct product of states on the two Hilbert spaces. Thus $|0, 0\rangle$ is entangled, while $|1, 1\rangle = |\uparrow\rangle_1 \otimes |\uparrow\rangle_2$ is not. Starting with a general state $|\psi\rangle = \sum_{i=1, j=1}^{d, d'} A_{ij} |\alpha_i\rangle_1 |\beta_j\rangle_2$,¹ one can use Schmidt's decomposition theorem [29] to show that there exists a basis for which one can write $|\psi\rangle = \sum_i \delta_{ij} \tilde{A}_i |\tilde{\alpha}_i\rangle_1 |\tilde{\beta}_i\rangle_2$ (the proof follows by rotating the α and β coordinates independently.) Using this basis, we can see that if we take the trace of the initial state $\rho = |\psi\rangle\langle\psi|$, over one Hilbert space, we obtain

$$\rho_1 = \sum_i \tilde{A}_i^* \tilde{A}_i |\tilde{\alpha}_i\rangle_1 \langle\tilde{\alpha}_i| \tag{2.1a}$$

or alternately,

$$\rho_2 = \sum_i \tilde{A}_i^* \tilde{A}_i |\tilde{\beta}_i\rangle_2 \langle\tilde{\beta}_i|. \tag{2.1b}$$

The mixed states associated with ρ_1 and ρ_2 , are the states seen by an observer restricting her measurements to part of the Hilbert space of states. Even though originally the entropy of the system was zero, if the state is entangled, the entropy seen by the observer will be non-zero. Further, as can be seen by eq. (2.1), the entanglement entropy measured by both observers is equal.

When measuring expectation values of an operator O_r that acts entirely within the Hilbert subspace \mathcal{H}_r , ($r = 1, 2$) one observes that [30]

$$\langle\psi|O_r|\psi\rangle = \sum_{i,j} \tilde{A}_i^* \tilde{A}_j \langle\tilde{\beta}_i|_1 \langle\tilde{\alpha}_i|O_r|\tilde{\alpha}_j\rangle_1 |\tilde{\beta}_j\rangle_2 \tag{2.2}$$

$$= \text{Trace}(\rho_r O_r) \quad (\text{no sum.}) \tag{2.3}$$

This implies that instead of considering expectation values of such operators in an entangled pure state, one may equivalently consider expectation values of the same operators in a mixed state.

Conversely, instead of considering expectation values of operators belonging to a Hilbert space \mathcal{H} in a mixed state $\rho = \sum_i |A_i|^2 |\alpha_i\rangle\langle\alpha_i|$, one can consider the same expectation values in a state $\psi = \sum_i A_i |\alpha_i\rangle |a_i\rangle$ defined on an extended Hilbert space $\mathcal{H} \otimes \mathcal{H}'$, where $|a_i\rangle$ is some arbitrary basis of \mathcal{H}' . If the density matrix ρ is to correspond to a thermal state in \mathcal{H} , then we should choose $|A_i|^2 = e^{-\beta E_i} / Z$ and $|\alpha_i\rangle = |E_i\rangle$, and E_i eigenvalues of the hamiltonian H associated with \mathcal{H} . Then we may define the pure state

$$|\psi\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta/2 E_i} |E_i\rangle |a_i\rangle. \tag{2.4}$$

¹In general, the dimensions of the two subspaces need not be equal. In this paper, we shall use Hilbert spaces of equal dimension.

The factor of two in the temperature appears since the weights of the thermal state are the squares of the amplitude of the entangled state.

If we further wish that the state defined in equation (2.4) be invariant under time translations, then may take the Hilbert space \mathcal{H}' to be a duplicate of \mathcal{H} , but take the global hamiltonian to be $H_{global} = H - H'$. The state obtained in this way

$$|\psi\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta/2 E_i} |E_i\rangle |E_i\rangle', \tag{2.5}$$

is called a thermofield double [31] (TFD). This procedure is also known as purification in the quantum information literature². We shall use thermofield doubles extensively in what follows.

Note that eq. (2.2) is valid regardless of the thermal nature of ρ and that one may create an entangled state to describe any ρ (and vice versa) using the above method. Thus, the thermal field double appears to be artificial. Remarkably, in some spacetimes, it turns out to be the natural physical state, as we shall now discuss.

3. Entanglement in spacetimes

3.1 Non-degenerate single horizon

Shortly after the discovery that black holes radiate, it was shown [25] that the vacuum of spacetimes with a single bifurcating Killing horizon is a TFD state, with respect to the left and right wedges of the spacetime. In this section we elaborate on these results, using a path integral method that was originally introduced in [26], showing that the natural structure of the Hilbert space of fields in a curved background with a bifurcating Killing horizon is a product of two Hilbert spaces, one for each wedge of the spacetime, and that the Hartle-Hawking vacuum state is the TFD state. This method offers a geometric interpretation of the doubling of the Hilbert spaces and allows an extension of the result to rotating and charged black holes.

We start from a metric of the form:

$$ds^2 = -f(r)d\tau^2 + dr^2/f(r) + q(r)dx_{\perp}^2, \tag{3.1}$$

with the radial coordinate restricted to the region $r \geq r_s$ ($f(r_s) = 0$.) The coordinate transformation

$$x = \sqrt{g(r)} \cosh(\tau f'(r_s)/2) \tag{3.2}$$

$$t = \sqrt{g(r)} \sinh(\tau f'(r_s)/2), \tag{3.3}$$

with

$$g = \frac{4\xi_0}{f'^2(r_s)} e^{f'(r_s) \int^r \frac{1}{f} dr} \tag{3.4}$$

²We thank D. Terno for pointing this out

gives us the metric

$$ds^2 = h(r)(-dt^2 + dx^2) + q(r)dx_{\perp}^2. \tag{3.5}$$

where $h(r) = \frac{1}{\xi_0} f(r) e^{-f'(r_s) \int^r \frac{dr'}{f(r')}}$ and ξ_0 is an integration constant. We can see explicitly that $h(r_s) \neq 0$ if $f(r)$ has a simple zero at r_s , and (assuming that the metric is analytic in x and t) that the coordinate patch now covers all of spacetime (except perhaps for a real singularity.)

In lorentzian signature, the metric in equation (3.5) covers all four wedges of the spacetime, while the metric in (3.1) covers one wedge. Therefore, we shall call the coordinates in (3.5) global coordinates, and the coordinates in (3.1) wedge coordinates. In euclidean signature the wedge coordinates cover all of euclidean spacetime.

Consider now a (scalar) field in the background of (3.5). Under the preceding assumptions, one may define [27] the Hartle-Hawking vacuum state $|0\rangle$ by switching to a euclidean signature, in which the wave functional of the vacuum is

$$\langle 0 | \psi(\vec{x}) \rangle = \int_{\varphi(\vec{x},0)=\psi(\vec{x})} \exp \left[- \int_0^\infty \dots \int \mathcal{L} d^d x d\tau \right] D\varphi,$$

where the integral is over all fields evaluated at $t > 0$ that satisfy the boundary conditions $\varphi(\vec{x}, 0) = \psi(\vec{x})$.

Next we define the density matrix

$$\rho_R = \text{Tr}_L |0\rangle \langle 0|,$$

where R, L correspond to the right and left wedges of spacetime. We wish to show that for the above spacetime $\rho_R = e^{-\beta H_R}$, where H_R is the wedge hamiltonian. We start by looking for a euclidean path integral representation of the elements of ρ_R , that is, of $\langle \psi'_R | \rho_R | \psi''_R \rangle$. In the specific case of interest,

$$\langle 0 | \psi_R \psi_L \rangle = \int_{\varphi(\vec{x},0)=\begin{cases} x>0,t=0 & \psi_R(\vec{x}) \\ x<0,t=0 & \psi_L(\vec{x}) \end{cases}} \exp \left[- \int_0^\infty \dots \int \mathcal{L} d^d x d\tau \right] D\varphi.$$

It follows that for $|\psi\rangle = |\psi_R \psi_L\rangle$, we have

$$\langle \psi' | \rho | \psi'' \rangle = \int_{\varphi(\vec{x},0)=\begin{cases} x>0,t=0^+, & \psi'_R(\vec{x}) \\ x>0,t=0^-, & \psi''_R(\vec{x}) \\ x<0,t=0^+, & \psi'_L(\vec{x}) \\ x<0,t=0^-, & \psi''_L(\vec{x}) \end{cases}} \exp \left[- \int_{-\infty}^\infty \dots \int \mathcal{L} d^d x d\tau \right] D\varphi. \tag{3.6}$$

Working in a euclidean signature, we may use the euclidean version of the wedge coordinates (3.1) to cover the entire spacetime instead of the global coordinates. This would be equivalent to changing from flat coordinates in the $x - t$ plane, to a ‘stretched’

(due to the $g(r)$ term in the transformation (3.2)) polar $r - \tau$ coordinate system. This will give us

$$\begin{aligned} x > 0, t = 0^\pm &\rightarrow r > r_s, \tau = 0^\pm \\ x < 0, t = 0^- &\rightarrow r > r_s, \tau = -\beta/2 \\ x < 0, t = 0^+ &\rightarrow r > r_s, \tau = \beta/2 \end{aligned}$$

so that in these coordinates equation (3.6) changes to

$$\langle \psi' | \rho | \psi'' \rangle = \int \varphi(\vec{x}, 0) = \begin{cases} r > r_s, \tau = 0^+, & \psi'_R(\vec{x}) \\ r > r_s, \tau = 0^-, & \psi''_R(\vec{x}) \\ r > r_s, \tau = \beta/2, & \psi'_L(\vec{x}) \\ r > r_s, \tau = -\beta/2, & \psi''_L(\vec{x}) \end{cases} \exp \left[- \oint \dots \int \mathcal{L} d^d x d\tau \right] D\varphi. \quad (3.7)$$

The integral over the τ coordinate is now on a circular path from $\tau = -\beta/2$ to $\tau = \beta/2$. Geometrically, this path integral is a product of two path integrals each over half a torus. One radial coordinate of the torus is the time coordinate, while the other specifies the spatial coordinates. The boundary conditions are such that the two half tori can not be glued together.

Now, the elements of ρ_R are given by

$$\begin{aligned} \rho_R &= \sum_{\psi_L} \langle \psi_L | \rho | \psi_L \rangle \\ &= \sum \langle \psi_L | \psi' \rangle \langle \psi' | \rho | \psi'' \rangle \langle \psi'' | \psi_L \rangle \end{aligned}$$

so

$$\langle \psi'_R | \rho_R | \psi''_R \rangle = \int \varphi(\vec{x}, 0) = \begin{cases} r > r_s, \tau = 0^+, & \psi'_R(\vec{x}) \\ r > r_s, \tau = 0^-, & \psi''_R(\vec{x}) \end{cases} \exp \left[- \oint \dots \int \mathcal{L} d^d x d\tau \right] D\varphi. \quad (3.8)$$

Now that the trace has been performed the two half tori have been glued at $\tau = \pm\beta/2$.

We shall show below that all the matrix elements of ρ_R are equal the matrix elements of $e^{-\beta H_R}$: $\langle \psi' | \rho | \psi'' \rangle = \langle \psi' | e^{-\beta H_R} | \psi'' \rangle$.

In the $r - \tau$ coordinates H_R , the generator of time translations, is time independent. So, we may use the standard time-slicing technique for expressing $e^{-\beta H_R}$ as a path integral in imaginary time from $\tau = 0$ to $\tau = -i\beta_0$,

$$\langle \psi'_{\text{in}} | e^{-\beta H_R} | \psi''_{\text{in}} \rangle = \int_{\substack{\varphi(x, 0) = \psi'_{\text{in}}(\vec{x}) \\ \varphi(x, -i\beta_0) = \psi''_{\text{in}}(\vec{x})}} \exp \left[i \int_0^{i\beta_0} \left(\int \pi \frac{d\varphi}{dt} d^d x - H_R \right) dt \right] D\varphi D\pi.$$

Changing to euclidean time:

$$\langle e^{-\beta H} \rangle = \int_{\substack{\varphi(x, 0) = \psi'_{\text{in}}(\vec{x}) \\ \varphi(x, \beta_0) = \psi''_{\text{in}}(\vec{x})}} \exp \left[\int_0^{\beta_0} \left(\int i\pi \dot{\varphi} d^d x - H \right) (d\tau) \right] D\varphi D\pi. \quad (3.9)$$

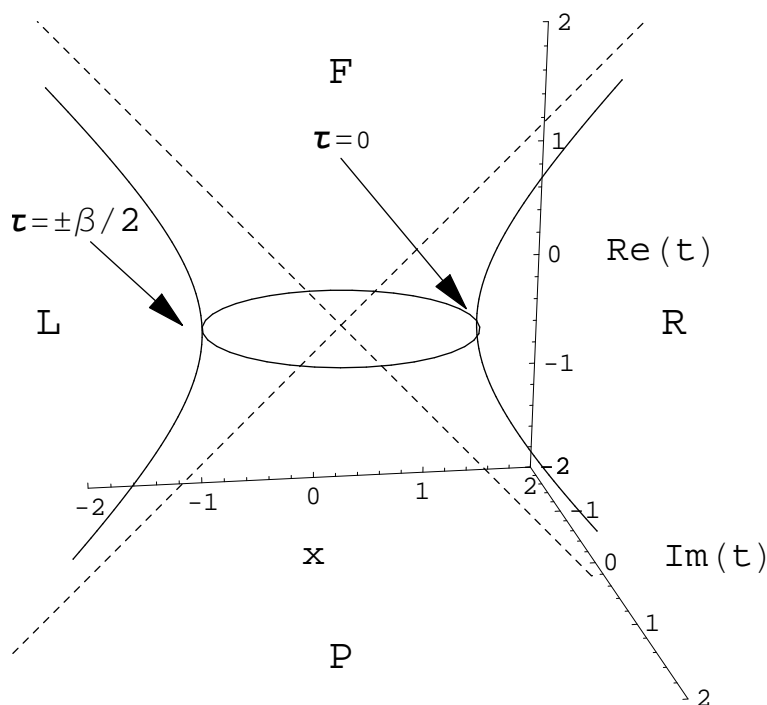


Figure 1: Diagram of a line of constant radial coordinate r in complex time. The circle is a line of constant r in imaginary time where τ is an angle, while the hyperbolas are lines of constant r in real time. The dashed lines represent, in real time, the bifurcation surface that splits the spacetime into the four wedges R, L, F, P.

Since H is the generator of time translations, it is the Legendre transform of a lagrangian. A lagrangian density for a scalar field in a gravitational background with metric $g_{\mu\nu}$ is given by $\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)$. The hamiltonian is $H = \int \mathcal{H}\sqrt{g}d^d x dt$, where $\mathcal{H} = \frac{1}{2}\left(\left(\frac{\pi}{g^{00}}\right)^2 + h^{ij}\partial_i\varphi\partial_j\varphi\right) + V(\varphi)$, h^{ij} are the space-space components of the metric ($h^{ij} = g^{ij}$), and $\pi = g^{00}\dot{\varphi}$.

Performing the gaussian integral and using $\Omega = \frac{1}{\sqrt{g_{00}^E}}$, (where $g_{00}^E = -g_{00}$), one obtains [32],

$$\langle \psi'_{\text{in}} | e^{-\beta H} | \psi''_{\text{in}} \rangle = \int_{\substack{\varphi(\xi,0)=\psi'_{\text{in}}(\xi) \\ \varphi(\xi,\beta_0)=\psi''_{\text{in}}(\xi)}} \exp \left[- \int_0^{\beta_0} \int \mathcal{L}_E d^d \xi d\eta \right] |\Omega| |g|^{\frac{1}{4}} D\varphi. \quad (3.10)$$

Obviously, equations (3.8), and (3.10) are equal up to the Liouville term.

We found that the matrix elements of $\text{Trace}_R |0\rangle\langle 0|$ are equal to those of $e^{-\beta H_R}$. From symmetry arguments, the same applies to $\text{Trace}_L |0\rangle\langle 0|$. It follows that the Hartle-Hawking state $|0\rangle$ is a TFD state.

Now that we know what the vacuum state looks like, we can find the entanglement entropy that is associated with the inaccessibility to the region of space beyond the horizon. This is given by $S_R = -\text{Tr}(\rho_R \log \rho_R)$. Since $\rho_R = e^{-\beta H_R}$, this is simply the standard

thermodynamical entropy associated with the fields in the wedge of spacetime, and was evaluated in [33]. In the high temperature limit, the entropy of a single scalar field in a curved background in $d + 1$ dimensions is given by $S = \frac{(d+1) T^d V_d}{\pi^{(d+1)/2}} \Gamma\left(\frac{d+1}{2}\right) \zeta(d+1)$, where V_d is the volume of space in the optical metric (obtained by a conformal transformation on the metric that leaves $g_{00} = 1$.) This entropy is formally infinite due to the divergence of the optical volume from the region near the horizon. Using (3.1) and setting $T = \frac{f'(r_s)}{4\pi}$ as the Hawking temperature, one finds

$$S = \frac{q(r_s)^{\frac{d-1}{2}} (d+1) \Gamma\left(\frac{d+1}{2}\right) \zeta(d+1)}{2^{2d-1} \pi^{\frac{3d+1}{2}} (d-1)} \delta^{-(d-1)} \int d\vec{x}_\perp. \tag{3.11}$$

The short distance cutoff away from the horizon, δ , is in units of proper distance. Had there been several fields that contribute to the entropy, and not only one scalar field, we would have found

$$S = \tilde{C} N_{\text{eff}} \delta^{-(d-1)} A_{\text{hor}}. \tag{3.12}$$

where N_{eff} takes into account the contributions of the various fields and \tilde{C} is a numerical factor. We see that the entanglement entropy is proportional to the surface of the horizon, and that it diverges as we allow the volume of optical space to enclose the regions of space that are infinitesimally close to the horizon surface. This ultra-violet (UV) divergence was first pointed out in [34]. We shall discuss this further in the context of the AdS/CFT correspondence in section 5.

3.2 Non-degenerate multiple killing horizons

For spacetimes with multiple Killing horizons, such as the charged or rotating black holes there are some slight subtleties: we consider a spacetime with the metric

$$ds^2 = -f(r)d\tau^2 + dr^2/f(r) + q(r)dx_\perp^2, \tag{3.13}$$

where now there are two horizons at $f(r_\pm) = 0$, where $f'(r_\pm) \neq 0$. An example would be the Reissner-Nordstrom solution for a charged black hole. If the inner horizon (r_-) is a cauchy surface for the observer at $r > r_-$, then one may consider only those four wedges on which the initial value problem for the field equation is well defined. In this case one can define a vacuum state, and follow through the procedure described above to show that the locally defined vacuum is the TFD of the left and right wedges for the observer at $r_- < r < \infty$. One should note however, that such a vacuum state is not well defined in all of spacetime [15, 35].

We proceed as in the previous section: the expression for ρ_R will be unchanged (eq. (3.8)). Though here the energy density: $E = T^{0\nu} K_\nu$ (where K_ν is the Killing vector for time translations and $T^{\mu\nu}$ is the energy momentum tensor), is not the same as the hamiltonian density, for a scalar field we find $\mathcal{H} = E + (ieA_0\pi\varphi + \text{c.c.})$, with π the canonical momentum. For an electrically charged black hole we may write this last term as $eA_0\sqrt{g}j^0$, where $j^\mu = i(\pi\phi - \pi^\dagger\phi^\dagger)$ is the scalar field charged current density. On comparing equations (3.8) and (3.10) we find

$$\rho_R = e^{-\beta E - \beta \int eA_0 j^0 \sqrt{g} d^d x}. \tag{3.14}$$

where the last term may be interpreted as a chemical potential.

We are unaware of a canonical form for the metric of rotating black holes. However, for all rotating black holes that we are aware of, it is possible to find a coordinate transformation similar to those in eq. (3.2) that extend the wedge geometry to the global one. An explicit example of such a transformation in the BTZ BH can be found in [36]. So, following the same considerations as before we expect that ρ_R for the case of rotating BH's is also given by eq. (3.8).

As opposed to the charged case, for rotating BH's $E = \mathcal{H}$. However, in the derivation of eq. (3.10) a complication arises: the time and angular directions are not orthogonal. This requires the extension of the standard path integral construction, since in the standard derivation one usually assumes that H will generate translations in the time direction only. If the hamiltonian generates spatial (or in this case, angular) translations as well as time translations we find that, for a scalar field, $\langle \psi(x', t') | \psi(x, t) \rangle = \int D\phi D\pi \exp \left(i \int \left(\int \left(\pi \dot{\phi} + N^i \partial_i \phi \pi \right) d^d x - H \right) dt \right)$ where $N^i = \frac{g^{0i}}{|g^{00}|}$ is the shift vector. This will then give us:

$$\rho_R = e^{-\beta E + \beta \int N^i \Pi_i d^d x} \tag{3.15}$$

where Π_i is the momentum in the i direction. The last term here may also be interpreted as a chemical potential. For a metric describing uniform rotation with angular velocity Ω , one has $N^\theta = \Omega$, so $N^i \Pi_i = \Omega L_\theta$, where L_θ is the angular momentum in the θ direction.

To summarize, we find that the non-extremal charged and rotating black holes have a vacuum structure similar to the non-rotating non-charged black hole. The partition function that is generated by tracing over the degrees of freedom in the adjacent spacetime wedge has a chemical potential, as expected on general grounds. Hence our results apply to rotating and charged BHs as well.

3.3 Extremal black holes

We define extremal black holes as spacetimes of the form (3.1) but here $f(r_s) = 0$ and $f'(r_s) = 0$. This may be viewed as a spacetime with multiple Killing horizons (3.13) where the limit $r_+ \rightarrow r_-$ has been taken. As the horizons approach each other, the throat (Einstein bridge) connecting the two left and right wedges seen by the $r > r_-$ observer becomes longer and longer, until it becomes infinitely long when $r_+ = r_-$ (see for example [37].) The equivalent of the HH state in this case is defined as the TFD state of the extremal black hole spacetime. We shall not discuss this case further.

4. Entanglement in field theory duals

Black hole entropy has been calculated in string theory using D-branes (originally, in ref. [8].) These provide a gauge theory dual to string theories in a curved background. The AdS/CFT is the best understood duality of this kind, and black hole entropy has been calculated in this setup as well [11]. In the AdS/CFT correspondence, the black hole entropy is related to the thermal entropy of the CFT at an appropriate temperature.

We relate such entropy calculations to entanglement entropy. To this end, we argue that the field theory dual of the black hole (global) vacuum is a TFD state (as originally

suggested in [16–18] in the context of the AdS/CFT correspondence.) This implies that the entropy of a thermal state in the FT may be interpreted as entanglement entropy and, as a consequence, that the black hole entropy may be interpreted as entanglement entropy. Thermal entropy may always be interpreted as coming from a fictitious extra Hilbert space. This was the original motivation for introducing a TFD [31]. However, for the case of black holes, this extra Hilbert space has geometric meaning — it is a reminder of the degrees of freedom beyond the horizon.

We start with a stringy description of a gravitational system in a background spacetime X , with a partition function $Z_{string}(X)$, and a field theory M with partition function $Z_{FT}(M)$. That the theories are dual, implies that $Z_{string}(X) = Z_{FT}(M)$. In the low energy ($\alpha' \rightarrow 0$), tree level ($g_s \rightarrow 0$), limit, the string theory partition function is approximately equal to the supergravity partition function $Z_{string} \approx e^{-I_{GRAVITY}}$. Under the AdS/CFT correspondence, this is dual to the strong coupling limit, $\lambda_{YM} \rightarrow \infty$, and $N \rightarrow \infty$ of the CFT. In what follows we shall assume that duality implies an isomorphism of the string and field theory Hilbert spaces. We assume that the isomorphism of Hilbert spaces should hold for any value of α' and g_s , including the low energy limit $\alpha' \rightarrow 0$, $g_s \rightarrow 0$.

Suppose that X is a spacetime with a bifurcating Killing horizon, and X_W is a wedge of this spacetime. Correspondingly, let M be the field theory dual of the string theory on X , and M_W be the field theory dual of the string theory on X_W . Let us denote the Hilbert space of small fluctuations of fields (including the gravitational field) around X by $\mathcal{H} = \mathcal{H}_W \otimes \mathcal{H}_W$. In the low energy limit, tracing over the degrees of freedom in the left wedge will result in a thermal state on the right wedge and vice versa. The partition function on the right wedge in this limit is $e^{-I_{GRAVITY}(X_W)} = Z(X_W)$, where the density matrix is given by $\rho_R = \text{Tr}_L |0\rangle\langle 0|$. So, one can go from $e^{-I(X)}$ to $e^{-I(X_W)}$ by tracing over states in the left wedge. We shall denote this tracing procedure as

$$e^{-I(X)} \xrightarrow{\text{Tr}} e^{-I(X_W)}. \tag{4.1}$$

As a result of the tracing procedure, the boundary conditions are different for the associated path integrals. The first has boundary conditions on the boundary of the extended space ∂X , while the second on the boundary of a wedge ∂X_W . The symbol $\xrightarrow{\text{Tr}}$ expresses the fact that to go from a partition function on the extended space X , $e^{-I(X)}$, to the partition function on a wedge X_W , $e^{-I(X_W)}$, we may take the state defining the first partition function, trace over half of its degrees of freedom, and evaluate the second partition function in a state defined by the resulting density matrix.

Let us consider the low energy limit of the dual field theory. The partition function is $Z_{FT}(M_W)$, and the density matrix is given by

$$\tilde{\rho}_W = \frac{1}{Z_{FT}} \sum_i e^{-\beta \tilde{E}_i} |\tilde{E}_i\rangle\langle \tilde{E}_i|. \tag{4.2}$$

In this low energy limit, the Hilbert space on the extended spacetime \mathcal{H} is isomorphic to the Hilbert space in the dual field theory $\tilde{\mathcal{H}}$, and \mathcal{H}_W is isomorphic to $\tilde{\mathcal{H}}_W$ so we must have $\tilde{\mathcal{H}} = \tilde{\mathcal{H}}_W \otimes \tilde{\mathcal{H}}_W$.

We observe that

$$Z_{FT}(M) = e^{-I(X)} \xrightarrow{\text{Tr}} e^{-I(X_W)} = Z_{FT}(M_W). \quad (4.3)$$

Therefore, $\tilde{\rho}_W$ is obtained from $\tilde{\rho}$ (the dual of $\rho = |0\rangle\langle 0|$) by an operation that is dual to tracing over the degrees of freedom in the left wedge of the vacuum state $|0\rangle\langle 0|$. The dual of tracing over the degrees of freedom in \mathcal{H}_W should be tracing over the degrees of freedom in $\tilde{\mathcal{H}}_W$. Let the dual of $|0\rangle$ be $|\tilde{0}\rangle$. Since the Hilbert space \mathcal{H}_W is isomorphic to $\tilde{\mathcal{H}}_W$ ($\mathcal{H}_W \cong \tilde{\mathcal{H}}_W$), it follows that for $\{|\psi_{L,i}\rangle\}$ a complete set in \mathcal{H}_W and $\{|\tilde{\psi}_{L,i}\rangle\}$ its dual in $\tilde{\mathcal{H}}_W$ that

$$\sum_i \langle \psi_{L,i} | 0 \rangle \langle 0 | \psi_{L,i} \rangle \cong \sum_i \langle \tilde{\psi}_{L,i} | \tilde{0} \rangle \langle \tilde{0} | \tilde{\psi}_{L,i} \rangle, \quad (4.4)$$

so

$$\rho_W = \text{Tr}_L |0\rangle\langle 0| \cong \text{Tr}_L |\tilde{0}\rangle\langle \tilde{0}|. \quad (4.5)$$

eq. (4.3) then reduces to $Z_{FT}(M) \xrightarrow{\text{Tr}} Z_{FT}(M_W)$. Therefore $\tilde{\rho}_W = \text{Tr}_L |\tilde{0}\rangle\langle \tilde{0}|$. Since tracing over one Hilbert space $\tilde{\mathcal{H}}_W$ of the state $|\tilde{0}\rangle$ results in a thermal state in the other, it follows that the $|\tilde{0}\rangle$ state is the TFD of the FT on $\tilde{\mathcal{H}}_W \otimes \tilde{\mathcal{H}}_W$.

We have now shown that the vacuum of the field theory M that is dual to string theory on the extended spacetime X is (in the low energy limit $\alpha' \rightarrow 0$, $g_s \rightarrow 0$) a TFD state. We have done this by appealing to the isomorphism of Hilbert spaces in the two descriptions, and to the special, entangled, nature of the HH vacuum. Let us now use this information to interpret black hole entropy as coming from entanglement. Suppose X_W is a spacetime whose entropy is $S(X_W) = \frac{A}{4G_N}$. Now consider the field theory dual M_W . Its (thermal) entropy should also be $S(M_W) = \frac{A}{4G_N}$. The entropy $S(M_W)$, as we have just shown, can be interpreted as due to entanglement in the initial TFD state $|\tilde{0}\rangle$ of M . This implies that $S(X_W)$ is also due to entanglement of the $|0\rangle$ state.

To summarize, we find a chain of equalities. First, the entanglement entropy on the gravity side is equal to the entanglement entropy on the FT side $S_{\text{entanglement, gravity}} = S_{\text{entanglement, FT}}$. Then, we know that the entanglement entropy on the FT side is equal to the thermal entropy of one FT at temperature $T > 0$: $S_{\text{entanglement, FT}} = S_{T>0, FT}$. Furthermore, we know that the thermal entropy of the FT is equal to the BH entropy $S_{T>0, FT} = S_{BH}$. The final result is that $S_{\text{entanglement, gravity}} = S_{BH}$.

Note that the entanglement entropy, defined at $G_N \rightarrow 0$, diverges as does the Bekenstein-Hawking entropy in this limit. To define and compare them at finite G_N one may use the dual theory, for which all quantities are well defined, as we have just done.

Referring to our earlier discussion surrounding eq. (3.12), consistency therefore requires a relationship between G_N , the number of light fields, and the short distance cutoff scale. There are several indications that such a relationship should exist. The first appearance of such a relation was in [34]. An indication of a similar relation using entropy bounds appeared in [38]. A different relationship between the maximal curvature of the universe, the number of light fields and the Planck scale, appears in [39–42]. More recently, such connections have been found in braneworld models [43, 44].

5. AdS black holes - an example

The entropy of the five dimensional AdS black hole in the wedge coordinates has been calculated either using the Hawking formula: $S_{BH} = \frac{A}{4G}$ or perturbatively in the CFT dual [11], which gives $S_{CFT} = 4/3 S_{BH}$. The different factor (4/3) arises because the Bekenstein-Hawking entropy and the CFT entropy have been calculated at two different points in the parameter space of the theory. Similar calculations for BH's in other dimensions were also performed [7].

Maldacena [18] has shown that the dual of the product theory $CFT \otimes CFT$ is approximately an AdS black hole in global coordinates. The approximation is related to contributions coming from sub-leading topologies. These were conjectured to have an important effect when considering Poincare recurrences, and were discussed in detail in [45, 46]. Here we will consider very massive AdS black holes for which the boundary is $\mathbb{R}^{n-1} \times S^1$ rather than $S^{n-1} \times S^1$ [47]. In this case there are no known contributions from other topologies to the partition function. In what follows, we shall therefore ignore contributions from sub-leading topologies.

A direct computation of the entanglement entropy of the AdS black hole in the low energy approximation ($\alpha' \rightarrow 0$) yields a divergent result (see eq. (3.12), and below) due to contributions from distances that are very close to the horizon.

Alternately, one may calculate the entanglement entropy of the AdS black hole via its CFT dual. In order to do this one should start off with the dual of the HH state $|\tilde{0}\rangle$, and trace over the Hilbert space that is dual to one of the wedges $\tilde{\mathcal{H}}_W$. The entanglement entropy seen by an observer, is the entropy associated with the state defined by $\tilde{\rho}_W = \text{Tr}_L |\tilde{0}\rangle\langle\tilde{0}|$. Since the state $|\tilde{0}\rangle$ is a TFD then $\tilde{\rho}_W = \frac{1}{Z} e^{-\beta \tilde{H}_W}$, so that the entanglement entropy is simply the entropy in a canonical ensemble of the CFT. By dimensional analysis this entropy is equal to $f(\lambda_{YM}) N^2 V T^3$. The function $f(\lambda_{YM})$ has been evaluated in [11], and should agree with the Hawking formula in the appropriate limit. Therefore, dualizing back to the AdS black hole, we find that the entanglement entropy agrees with the Bekenstein-Hawking formula.

To make the comparison in detail, let us consider the case of the 5D AdS BH. Recall that for large black holes in 5D AdS the Einstein frame metric is approximately give by

$$ds^2 = - \left(\frac{r^2}{R^2} - \frac{b^4 R^2}{r^2} \right) dt^2 + \left(\frac{r^2}{R^2} - \frac{b^4 R^2}{r^2} \right)^{-1} dr^2 + \frac{r^2}{R^2} (dx_1^2 + dx_2^2 + dx_3^2). \quad (5.1)$$

The constant $b \gg 1$ is related to the 5D Newton's constant G_5 , to the mass of the BH M_{BH} , to the AdS scale R , $b^4 = \frac{8G_5}{3\pi} \frac{M_{BH}}{R^2}$, and to the BH temperature $b \simeq \pi R T_{BH}$. The BH horizon is at $r_s \simeq bR$. The entropy of the BH is given by

$$S_{BH} = \frac{A_{\text{hor}}}{4G_5} = C \frac{R^5 A_{\text{hor}}}{l_s^8 g_s^2}, \quad (5.2)$$

where C is a convention dependent numerical factor that determines the relationship between the string length and Newton's constant in 10D and l_s is the string scale. The area of the horizon is given by $A_{\text{hor}} = \frac{r_s^3}{R^3} \int dx_1 dx_2 dx_3$.

As we have discussed, the entanglement entropy $S_{\text{entanglement, gravity}}$ in eq. (3.12) is equal to the Bekenstein-Hawking entropy S_{BH} in eq. (5.2). This gives the following relation

$$\tilde{C} N_{\text{eff}} \delta^{-3} = C \frac{R^5}{l_s^8 g_s^2}. \tag{5.3}$$

Equation (5.3) may be justified as follows. Recall that we are using the low energy approximation, and that the 5D metric is only a part of the full 10D metric that describes a BH in $AdS_5 \times S^5$. In order for the low energy approximation to be valid, we need that massive string modes do not contribute to the action. This gives, in the Einstein frame, $\delta \sim g_s^{1/4} l_s = l_p$ (where l_p is the Planck length.) This cutoff scale is much higher the Kaluza-Klein scale. Consequently we need to include in N_{eff} the full tower of Kaluza-Klein modes on the five sphere. For each 10D massless field we get $N_{\text{eff}}^{(5)} \sim \frac{R^5}{\delta^5}$ fields in 5D, hence $N_{\text{eff}} = K \frac{R^5}{\delta^5}$ (K is a numerical factor). Substituting the expressions for N_{eff} and δ into $N_{\text{eff}} \delta^{-3}$ we obtain eq. (5.3). This analysis generalizes to higher dimensional black holes.

We have compared the bulk theory provided with a short distance cutoff to a theory with finite N and λ_{YM} on the boundary. The short distance cutoff and finite N were used to regulate the entropy. Since currently we do not know of a precise relation between the two regularization prescriptions, we can only obtain a relationship between the entanglement entropy and the Bekenstein-Hawking entropy up to an undetermined numerical factor.

6. Discussion

Due to the structure of the Hilbert space of states of fields in a spacetime background that admits a bifurcating Killing horizon, an observer in part of the spacetime will observe a density matrix reflecting her lack of information regarding the region beyond the horizon. For thermal states, the associated entropy is called entanglement entropy. From this point of view, for eternal BHs, the entanglement entropy is the entire black hole entropy, in contrast to some of the previous interpretations where entanglement entropy was considered to be a quantum correction to the black hole entropy [19–23], and in agreement with the “induced gravity” approach [24].

Several properties of black holes that are usually considered strange may be reinterpreted in light of this interpretation. The area scaling of the entropy or of other thermodynamic quantities is a property that is generic to spatially entangled systems. In an entangled state, the entanglement entropy of the two spatial regions must be equal, and this is suggestive that it be proportional to the mutual surface area. A more thorough analysis shows that the area dependence comes from the properties of short range correlations near the space boundary [48–52].

If there is an interaction term in the hamiltonian of the two wedges (which may come from the kinetic terms for the fields when putting spacetime on a lattice), then the entangled nature of the vacuum will become physically relevant: when restricting an observer to one wedge, information will seem to “leak” into the other wedge, and so the evolution will be non-unitary. This argument may be made more precise and will appear in [53].

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